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Entropy and Temperature of Black 3-Branes

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Abstract

We consider slightly non-extremal black 3-branes of type IIB supergravity and show that their Bekenstein-Hawking entropy agrees with the counting of states of the Dirichlet 3-brane. The Dirichlet brane excitations are described in terms of the statistical mechanics of a 3+1 dimensional gas of massless open string states. This is essentially the classic problem of black-body radiation. The blackbody temperature is shown to be identical with the temperature of the Hawking radiation. We also construct a solution of type IIB supergravity describing a 3-brane with a finite density of longitudinal momentum. For extremal momentum-carrying 3-branes the horizon area vanishes. This is in agreement with the fact that the BPS entropy of the momentum-carrying Dirichlet 3-branes is not an extensive quantity.

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1 Introduction

Apart from their intrinsic importance, black holes[4] provide a testing ground for the quantum theory of gravitation. Classical General Relativity, together with quantum field theory, implies that a black hole should be assigned an entropy equal to one-fourth of its horizon area measured in Planck units [1, 2]. In a fundamental theory of quantum gravity this Bekenstein-Hawking entropy should have a statistical interpretation. It has been argued [3, 4, 5] that string theory provides such an interpretation, because very massive fundamental string states should form black holes, and the number of such states exhibits the exponential Hagedorn growth.

Recently, a much improved understanding of the Ramond-Ramond charged string solitons has emerged through the Dirichlet brane description [6, 7]. This has led to rapid progress on the black hole entropy problem. In [8] a certain extremal 5-dimensional black hole was constructed so that its horizon area is non-vanishing. It was shown that the logarithm of its ground state degeneracy, calculated with D-brane methods, precisely matches the Bekenstein-Hawking entropy. This remarkable finding has been extended in a number of directions. In [9] it was generalized to rotating black holes. In [10] a similar 5-dimensional example was considered, and it was further shown that the entropy of slightly non-extremal black holes also matches the Bekenstein-Hawking result. This allowed for a D-brane calculation of the temperature of Hawking radiation. In [11] similar results were obtained for slightly non-extremal black strings in 6 dimensions (upon compactification these strings reduce in a certain limit to the 5-dimensional black holes of [8]).

At this stage it is important to elucidate the criteria for agreement between the D-brane and the Bekenstein-Hawking entropy, and to find new successful examples. In this paper we provide a new and very simple example of a black p-brane whose D-brane entropy matches the Bekenstein-Hawking entropy. This is the self-dual 3-brane in 10 dimensions. Since it couples to the self-dual 5-form, it automatically carries equal electric and magnetic charge densities. A special property of this object, as well as of those in [8]–[11], is that the string coupling is independent of position. Control over the value of the string coupling at the horizon appears to be necessary for agreement between the two definitions of entropy. For p-branes with $p < 3$ it is easy to check that the D-brane entropy is not proportional to the horizon area. This is likely due to the string coupling becoming strong near the p-brane.

The original 3-brane solution of type IIB supergravity was constructed in [12]. In section 2 we observe that at extremality this solution has vanishing horizon area. We construct a new class of solutions describing 3-branes carrying finite momentum density along one of its internal dimensions. Although the longitudinal momentum is known to stabilize the horizon area of extremal black strings [11], here we find that it does not. The fact that the classical entropy is zero agrees with the fact that the logarithm of the ground state degeneracy of the momentum-carrying Dirichlet 3-branes is not an extensive quantity. In order to address

[4] In this short note we will not attempt to reference all of the developments in the recent black hole literature.

objects with non-vanishing horizon area, in section 3 we consider slightly non-extremal 3-branes, whose masses satisfy $\delta M = M - M_0 \ll M_0$. To leading order in the parameter $\delta M/M_0$, which is a measure of deviation from extremality, we find agreement between the D-brane entropy and 1/4 of the horizon area. Amusingly, the statistical mechanics of a non-extremal 3-brane is that of a photon (and photino) gas in 3 + 1 dimensions, which is the classic blackbody radiation problem. The scaling of entropy with energy may be derived essentially from the well-known blackbody scaling laws,

$$M - M_0 \sim VT^4 \ , \qquad\qquad S \sim VT^3 \ . \tag{1}$$

Working out the overall factor in the Bekenstein-Hawking relation forces us to the surprising conclusion that only the transverse excitation modes of the 3-brane should be counted in determining the entropy. The internal (longitudinal) degrees of freedom are somehow required to be absent from the counting.

Upon coupling of the 3-brane to the 10-dimensional world, waves colliding on the 3-brane may be converted to massless closed string states. This is Hawking radiation in the D-brane language [10]. The blackbody temperature that one assigns to a non-extremal 3-brane acquires the interpretation of the Hawking temperature. In section 4 we conclude with a brief discussion.

2 Entropy of 3-branes carrying longitudinal momentum

The 3-brane solution to the equations of type IIB supergravity was originally obtained by Horowitz and Strominger [12] and is given by

$$\begin{aligned} ds^2 &= -\Delta_+\Delta_-^{-1/2}dt^2 + \Delta_-^{1/2}(dx_1^2 + dx_2^2 + dx_3^2) + \Delta_+^{-1}\Delta_-^{-1}dr^2 + r^2 d\Omega_5^2 \\ F_{(5)} &= Q(\varepsilon_5 + *\varepsilon_5) \\ \Phi &= \text{const}\,. \end{aligned} \tag{2}$$

In these equations $F_{(5)}$ is the Ramond-Ramond self-dual 5-form field strength coupling to the 3-brane, and the dilaton field has an arbitrary constant value for this solution. We have also defined

$$\Delta_\pm(r) = \left(1 - \frac{r_\pm^4}{r^4}\right) \ . \tag{3}$$

The charge density on the 3-brane is

$$Q = 2r_+^2 r_-^2 \equiv 2r_0^4 \tag{4}$$

up to a convention-dependent proportionality constant. In this section we will ignore such constants since they are irrelevant to our calculations. For the solution to be well-behaved, we need $r_+ \geq r_-$. Extremality is achieved when the horizon radius r_+ becomes equal to r_-. The extremal ADM mass is proportional to Q, as required by supersymmetry. The extremal solution preserves one-half of the ten dimensional type IIB supersymmetries, i.e. $N = 1$. We also introduce an infrared cut-off by compactifying each internal coordinate x^i on a very large circle of radius L, i.e. imagine that the 3-brane is wrapped around a large 3-torus T^3.

The 8-dimensional area of the horizon is

$$A = \omega_5 r_+^5 L^3 \left[\Delta_-(r_+)\right]^{3/4} \ , \tag{5}$$

where $\omega_5 = \pi^3$ is the area of a unit 5-sphere. The classical black 3-brane entropy,

$$S_{BH} = \frac{A}{4} \,, \tag{6}$$

therefore vanishes in the extremal limit.

If we fix the charge and consider a slightly non-extremal black 3-brane then, as we will see in the next section, the entropy of the classical extremal black 3-brane scales as

$$S_{ext} \sim \omega_5 L^3 r_0^5 \left[\frac{\delta M}{M_0}\right]^{3/4} \ . \tag{7}$$

In the case of the black string [12], which also had zero area at extremality, it was possible to perform a boost along the string to induce simultaneously finite ADM momentum and horizon area.

It is also easy to inject momentum P along one[5] of the three spatial worldbrane directions, which we take to be x^1. The appropriate solution may be found by performing a (now-standard) boost on the solution (2). In this way we obtain

$$\begin{aligned} ds^2 &= -\left(\cosh^2\alpha \Delta_+ \Delta_-^{-1/2} - \sinh^2\alpha \Delta_-^{1/2}\right) dt^2 \\ &+ \left(\cosh^2\alpha \Delta_-^{1/2} - \sinh^2\alpha \Delta_+ \Delta_-^{-1/2}\right) dx_1^2 \\ &+ \sinh(2\alpha)\left(\Delta_-^{1/2} - \Delta_+ \Delta_-^{-1/2}\right) dt dx_1 \\ &+ \Delta_-^{1/2}(dx_2^2 + dx_3^2) + \Delta_+^{-1}\Delta_-^{-1} dr^2 + r^2 d\Omega_5^2 \,. \end{aligned} \tag{8}$$

If we imagine that the T^3 is small, then we can think of the configuration (8) as a seven-dimensional black hole. The black hole has a gauge charge corresponding to the gauge field which comes from the (t, x^1) cross term in the metric. Note that this extremal solution is still BPS-saturated, as it preserves one supersymmetry of a possible four (type IIB compactified on T^3 to $d = 7$ has $N = 4$ supersymmetry). In ten dimensional language this "charge" is just the total ADM momentum, which is given by

$$\begin{aligned} P_{ADM} &= \frac{L^3\omega_5}{8\pi}\sinh(2\alpha)(r_+^4 - r_-^4) \\ &\equiv \frac{2\pi n}{L} \,, \end{aligned} \tag{9}$$

[5] Note that our conclusions would be unchanged if we performed additional boosts involving any of the other spatial worldbrane directions.

where n is an integer and we are keeping the ten dimensional Newton constant fixed.

If we let the deviation from extremality go to zero, but also take the limit of infinite boost parameter, then for finite ADM momentum

$$P_{ADM} \sim L^3 \omega_5 Q \left[e^{2\alpha} \frac{\delta M}{M_0} \right] , \tag{10}$$

we need the scaling $\delta M / M_0 \sim e^{-2\alpha}$.

Then the entropy of a BPS-saturated state with this momentum number n is finite and given by

$$\begin{aligned} S_{BPS} &\sim 2\pi\sqrt{2n} \\ &\sim L^2 \left[\omega_5 Q\right]^{1/2} \left[\frac{\delta M}{M_0} e^{2\alpha} \right]^{1/2} . \end{aligned} \tag{11}$$

This quantity is not extensive in the spatial worldvolume of the 3-brane. The entropy density, measured per unit spatial worldvolume, goes as

$$s_{BPS} \equiv \frac{S_{BPS}}{L^3} \to 0 \,. \tag{12}$$

For a Dirichlet p-brane, this zero BPS entropy will actually happen for any value of $p > 1$, as follows. A BPS-saturated excitation on the worldvolume is effectively restricted to live in a single dimension, because if there were two finite orthogonal momenta then the state would no longer be BPS-saturated. Therefore the scaling goes as $S_{BPS} \sim \sqrt{n}$, while $P_{ADM} \sim L^p$, so that $n \sim L^{p+1}$, and therefore

$$s_{BPS} \sim L^{(p+1)/(2p)} \to 0 \,. \tag{13}$$

So we see that in order to have finite, nonzero ADM momentum and finite, nonzero entropy, both measured per unit spatial worldvolume, we need $p = 1$, i.e. the string.

Let us now compare this conclusion about the Dirichlet 3-brane entropy with results for the classical black 3-brane configuration. Due to the boost, we find that the Bekenstein-Hawking entropy of the classical configuration (2) is altered from its previous value to

$$\begin{aligned} S_{BH} &= \frac{\omega_5}{4} r_+^5 L^3 \left[\Delta_-(r_+)\right]^{3/4} \cosh\alpha \\ &\sim \omega_5 L^3 r_0^5 \left[\frac{\delta M}{M_0} \right]^{3/4} e^{\alpha} \end{aligned} \tag{14}$$

as $\alpha \to \infty$ and $\delta M / M_0 \to 0$. Let us now take the limit such that the ADM momentum remains finite. Then we need the scaling $\delta M / M_0 \sim e^{-2\alpha}$ and so the classical 3-brane area goes as

$$A \sim e^{-3\alpha/2} e^{\alpha} \to 0 \,. \tag{15}$$

This tells us that the BPS-saturated 3-brane with finite nonzero momentum still has zero area. Note that if we consider a modified area given by the classical horizon area divided by

$\sqrt{g_{22}(r_+)g_{33}(r_+)}$, this scales similarly to the quantity (11); however, it is difficult to give this modified area an enlightening physical interpretation. [6]

Therefore we see that the entropy of the BPS-saturated classical 3-brane with momentum, which by definition is extensive in the horizon area, is also zero. It is satisfying that the entropies on the classical black 3-brane and Dirichlet 3-brane sides agree, as expected.

3 Statistical Mechanics of Non-extremal 3-branes

In this section we will consider non-BPS excitations of the 3-brane. In the D-brane picture the excitations we have in mind are described by a dilute gas of massless open string states running along the brane in arbitrary directions. The average total momentum is zero. The momenta of the massless string states are quantized:

$$\vec{p} = \frac{2\pi}{L}\vec{n} \tag{16}$$

where $\vec{n} \in \mathbf{Z}^3$. The mass of the excited 3-brane is

$$M = M_0 + \delta M = \frac{\sqrt{\pi}}{\kappa}L^3 + \sum_{i=1}^{k} \frac{2\pi}{L}|\vec{n_i}| + O(g) \ . \tag{17}$$

Here M_0 is the mass of the extremal 3-brane [13], k is the number of open strings, and

$$\kappa = \sqrt{8\pi G_N} = g\alpha'^2 \ . \tag{18}$$

The $O(g)$ term in (17) accounts for interactions among the strings. The validity of counting these states and no others to obtain the entropy of a non-extremal p-brane was discussed in [14] for the case $p = 1$, and the same arguments apply here. In particular, our ability to control the decay rate of the non-BPS states by making L large allows us to count these states reliably with g and hence G_N finite.

Rather than calculating the degeneracy of excited 3-brane states at a given δM directly, let us instead consider the statistical mechanics of massless open string states in the grand canonical ensemble. The temperature T will later be identified as the Hawking temperature, but for now one can regard our ensemble calculations as a trick to figure out the degeneracies of brane excitation levels.

Writing down the correct partition function turns out to be more subtle than one might first expect. We claim that the correct answer is

$$Z = \prod_{\vec{n}\in\mathbf{Z}^3} \left(\frac{1+q^{|\vec{n}|}}{1-q^{|\vec{n}|}}\right)^6 \tag{19}$$

[6]Note also that in the above scaling limit g_{tt} diverges on the horizon. We thank Gary Horowitz for pointing this out to us.

where we have defined

$$q = e^{-2\pi/LT} \ . \tag{20}$$

Naively one would expect the exponent in (19) to be 8, not 6, since there are eight massless bosons and eight massless fermions in the open string spectrum. The dynamics of these modes on the brane is given by $N = 4$ supersymmetric pure Yang-Mills theory with gauge group $U(1)$ [15, 16, 17]. For our purposes, however, it is more revealing to view this theory as $N = 1$ Yang-Mills plus six chiral multiplets. The chiral multiplets are associated with transverse oscillations of the brane, while the gauge multiplet describes internal degrees of freedom. To obtain agreement with the Bekenstein-Hawking entropy it seems necessary to count only the modes of transverse oscillation, hence the exponent 6 in (19).

What subtlety of the gauge dynamics might prevent the gauge degrees of freedom from being enumerated along with the transverse oscillations? A. Tseytlin has suggested to us the following interesting mechanism [18]. If one imposes periodic boundary conditions on the gauginos along the Euclidean time direction rather than the standard antiperiodic boundary conditions, then the two physical gaugino degrees of freedom introduce a factor $(1-q^{|\vec{n}|})^2$ into the partition function, exactly cancelling the gauge boson contribution, $(1 - q^{|\vec{n}|})^{-2}$. Thus the gauge dynamics becomes in effect topological. We look forward to exploring possible justifications and consequences of this insightful guess for the gaugino boundary conditions.

Equation (19) includes six physical bosonic and six physical fermionic modes, and in $3+1$ dimensions each fermion mode makes $7/8$ the contribution of a boson mode to the entropy and energy (the corresponding ratio in $1 + 1$ dimensions is $1/2$). Using the relations

$$\begin{array}{c} F = -T \log Z \\ E = T^2 \frac{\partial}{\partial T} \log Z \\ S = (E - F)/T \end{array} \tag{21}$$

we find

$$\begin{aligned} E &= \frac{3\pi^2}{8} L^3 T^4 \\ S &= \frac{\pi^2}{2} L^3 T^3 \ . \end{aligned} \tag{22}$$

At this point it is easy to see how things change when n_w 3-branes are stacked on top of one another. The massless open strings can now connect any two of the branes, so there are n_w^2 states for every one state we had before. In this context it is important to recall that there is no binding energy among the 3-branes [15], so strings running between different branes really are massless. Furthermore, when L is large, it makes no difference whether we consider n_w singly wound branes or one brane wrapped n_w times around T^3: the asymptotic density of massless string states per unit volume is unaffected by such changes in boundary conditions.

To recapitulate, the prescription for $n_w > 1$ is to consider n_w^2 (very weakly) coupled thermodynamic systems, each identical to the $n_w = 1$ system treated above. Thus (22) becomes

$$\begin{aligned} E &= \frac{3\pi^2}{8} n_w^2 L^3 T^4 \\ S &= \frac{\pi^2}{2} n_w^2 L^3 T^3 \ . \end{aligned} \tag{23}$$

The relation between E and S in the microcanonical ensemble is determined by eliminating T from (23):

$$S = 2^{5/4} 3^{-3/4} \sqrt{\pi n_w} L^{3/4} E^{3/4} \ . \tag{24}$$

Setting $E = \delta M$ in (24), one obtains the entropy of non-extremal 3-branes with mass $M_0 + \delta M$. Using the formula [13]

$$M_0 = \frac{\sqrt{\pi}}{\kappa} n_w L^3 \tag{25}$$

one can show finally that

$$S = 2^{5/4} 3^{-3/4} \pi^{7/8} n_w^{5/4} \kappa^{-3/4} L^3 \left(\delta M / M_0\right)^{3/4} \ . \tag{26}$$

This expression for S should be comparable to the Bekenstein-Hawking entropy. Let us therefore turn to the calculation of the horizon area in the low-energy supergravity theory.

The ADM mass formula for the black 3-brane described by the metric (2) is [19]

$$M_{ADM} = \frac{\omega_5 L^3}{2\kappa^2} (5 r_+^4 - r_-^4) \ . \tag{27}$$

Applying this formula to the extremal case $r_+ = r_- = r_0$ and comparing with (25), one finds

$$r_0^4 = \frac{\sqrt{\pi}}{2\omega_5} n_w \kappa \ . \tag{28}$$

The RR charge remains unchanged as we perturb away from extremality, so $r_- = r_0^2 / r_+$. Writing $r_+ = r_0 + \varepsilon$, one finds from (27) that

$$\frac{\delta M}{M_0} = 6 \frac{\varepsilon}{r_0} \tag{29}$$

to lowest order in ε. Thus the horizon area of the metric (2) is

$$\begin{aligned} A &= \omega_5 r_+^5 L^3 \left(1 - \frac{r_-^4}{r_+^4}\right)^{3/4} \\ &= 2^{9/4} \omega_5 r_0^5 L^3 \left(\frac{\varepsilon}{r_0}\right)^{3/4} \\ &= 2^{1/4} 3^{-3/4} \pi^{-1/8} (n_w \kappa)^{5/4} L^3 \left(\delta M / M_0\right)^{3/4} \end{aligned} \tag{30}$$

and the Bekenstein-Hawking entropy is

$$S_{BH} = \frac{2\pi A}{\kappa^2} = 2^{5/4} 3^{-3/4} \pi^{7/8} n_w^{5/4} \kappa^{-3/4} L^3 \left(\delta M / M_0\right)^{3/4} \tag{31}$$

in exact agreement with (26)!

A bonus we get for computing the entropy in the grand canonical ensemble is that the blackbody temperature T used in (19-23) is precisely the Hawking temperature. This is a trivial consequence of the relation $M = M_0 + E$ where E is the energy of the gas of massless open strings. We know from ordinary statistical mechanics that $dE = TdS$ when L is held fixed. But $dE = dM$, so the relation $dM = T_H dS_{BH}$ from black hole thermodynamics leads immediately to

$$T_H = T = \left(\frac{8}{3\pi^2 n_w} \frac{\delta M}{L^3} \right)^{1/4} . \tag{32}$$

At first it seems surprising that the Hawking temperature should be independent of the string coupling g. But it becomes inevitable when one realizes that $T_H = T$, since the properties of the dilute gas of open string states characterizing the excitation of the D-brane depend in no way on g. The string coupling determines only the degree of diluteness necessary to make our arguments valid. It remains a fascinating problem to derive this g-independent temperature from a string perturbative calculation of the amplitudes for decay processes of the excited 3-brane, similar to the scattering amplitudes computed in [21].

Note that if we had included all eight bosonic and fermionic modes in the statistical mechanics treatment of D-brane excitations, we would have obtained an entropy [20]

$$S = \left(\tfrac{4}{3}\right)^{1/4} S_{BH} . \tag{33}$$

A compensating factor would then have to be introduced into (32) if the thermodynamic relation $dM = T_H dS_{BH}$ is to be preserved.

4 Discussion

In this paper we have presented a very simple Dirichlet brane system whose entropy is reproduced by the Bekenstein-Hawking entropy of the corresponding low-energy supergravity solution. This agreement, not only in the scaling with respect to all parameters but in the coefficient as well, is so miraculous that it clearly requires a deeper understanding. How does classical type IIB supergravity "know" the Planck formula for blackbody spectrum? Apparently it does. And what are we to make of the necessity of excluding the gauge modes from the state-counting calculation of D-brane entropy? In view of the success of the suggestion [18] to make the gaugino fields periodic in Euclidean time, we would view the Bekenstein-Hawking result as a hint of topological gauge dynamics on the brane. The precise role of the worldvolume gauge field needs to be explored in detail.

Motivated by [10] we would also like to show precisely how the 3-brane blackbody temperature translates into the Hawking temperature of the outgoing closed string radiation. We hope to report on these issues in the near future.

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